Uncertainty Estimation for Ensemble Particle Image Velocimetry

A. Ahmadzadegan[1*], S. Bhattacharya[1], A. M. Ardekani[1], P. P. Vlachos[1]

[1] School of Mechanical Engineering, Purdue University, West Lafayette Indiana, 47907, USA

[*] aahmadza@purdue.edu

Abstract

We present a novel approach to estimate the uncertainty in ensemble particle image velocimetry (PIV) measurements. Ensemble PIV is widely used when the cross-correlation signal-to-noise ratio (SNR) is insufficient to perform a reliable instantaneous velocity measurement. Despite the utility of ensemble PIV, uncertainty quantification for this type of measurement has not been studied. The existing uncertainty quantification algorithms for PIV are developed and used only for instantaneous PIV measurement and do not account for the improved SNR in ensemble PIV. Existing instantaneous uncertainty quantification methods can be divided into direct and indirect categories. Indirect methods require calibration based on the effect of various image parameters (such as noise, particle size, density, velocity gradient, etc.) on the correlation SNR. Indirect methods have not been calibrated for error sources relevant in an Ensemble PIV measurement. Also, they have lower sensitivity to the error sources compared to direct approaches. Direct methods, such as the moment of correlation (MC) and Image Matching (IM), find the uncertainty based on the images and correlation planes without any calibration and are more reliable (Bhattacharya et al., 2018; Sciacchitano et al., 2013). Ensemble PIV is based on ensemble correlations; therefore, MC, which uses the generalized cross-correlation (GCC) plane as a measure of uncertainty, is the most suitable method to be modified to be applicable for the ensemble PIV. The GCC plane is the inverse Fourier transform of the phase correlation and represents the probability density function (PDF) of particles' displacements (Bhattacharya et al., 2018; Eckstein and Vlachos, 2009). We replaced instantaneous GCC with ensemble GCC and modified MC's normalization factor to account for the number of ensembles. The MC's primary limitation is that it assumes a Gaussian shape for the PDF of displacements and estimate the standard deviation of the underlying PDF using a fitted Gaussian. However, the PDF deviates from Gaussian distribution due to velocity gradient or non-Gaussian random displacements. Therefore, MC's reliability and applicability are reduced for flow fields with non-Gaussian PDFs. Also, our analysis shows that ensemble MC consistently underestimates the uncertainty. So, a generalized and reliable method for uncertainty quantification for ensemble PIV is needed.

To address this gap, we developed a method termed as Moment of Probability of Displacement (MPD). We base our approach on directly estimating the PDF of particles' displacement using the image-based probability estimation of displacement (iPED) (Ahmadzadegan et al., 2020). This method finds the PDF by deconvolving the ensemble autocorrelation from the ensemble cross-correlation as shown in eq.1.

$$PDF = \mathcal{F}^{-1}(\mathcal{F}(CC_e)/\mathcal{F}(AC_e)) \tag{1}$$

We then calculate the second moment (standard deviation) of the estimated PDF (σ_{PDF}). For calculating σ_{PDF}, unlike ensemble MC, we do not use any model to represent the PDF. Instead, we calculate the σ_{PDF} in a discrete manner. To report the uncertainty of velocity measurement, σ_{PDF} needs to be scaled down by the square root of the number of samples contributing to the distribution to be consistent with the definition of standard error. Therefore, the uncertainty in velocity measurements (σ_{MPD}) can be found using the normalization factor shown in eq. 2 that accounts for the number of image ensembles (N) and the average mutual information (MI) between the image pairs (Xue et al., 2015).

$$\sigma_{MPD} = \sigma_{PDF}/\sqrt{(MI \times N)} \tag{2}$$

We assess MPD's performance by running a Monte Carlo analysis using synthetic images designed to study the response of MPD to different elemental PIV error sources, namely, particle size, seeding density, amount

of displacement randomness (diffusion coefficient), mean displacement, background noise, Shear, and the number of image ensembles. It has been established that a reliable uncertainty quantification algorithm predicts uncertainties that match the expected root mean square (RMS) of the error in the velocity measurements (Sciacchitano et al., 2015). Fig. 1 shows the RMS of predicted uncertainty using MPD and ensemble MC compared to RMS of the error for each primary error source. We show that in all cases, MPD predicted uncertainties that follow the RMS error trend. So, MPD shows good sensitivity to the elemental PIV error sources with around %86 accuracy in matching with the RMS of error in the baseline data sets. MPD also outperforms the ensemble MC which consistently underestimates the uncertainty.

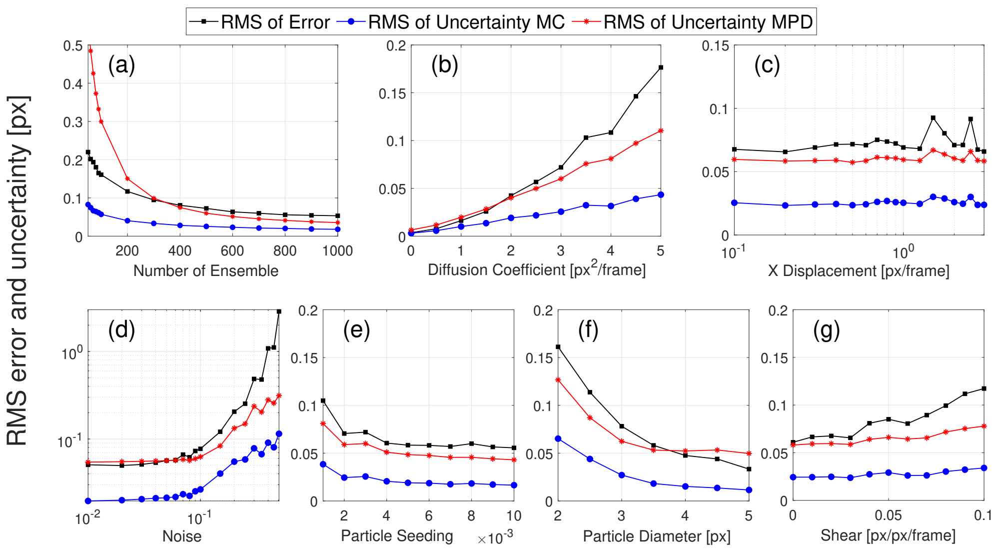

Figure 1: The sensitivity of MPD and ensemble MC compared to elemental error sources. The RMS of error (black lines) compared to RMS of uncertainty using MPD (red lines) and ensemble MC (Blue lines) with respect to a) number of ensembles, b) diffusion coefficient, c) particle displacement, d) background noise, e) particle seeding density, f) particle diameter, g) shear rate.

Subsequently, we demonstrate the performance of MPD on experimental microscopy images of flow in a rectangular micro-channel. In Fig. 2 we show that uncertainty and absolute error distributions. The horizontal lines show the RMS for each distribution. We show that the RMS of uncertainty from MPD agrees well with the RMS of the error while outperforming the ensemble MC.Thus, the proposed methodology shows good sensitivity to the RMS error over a range of signal to noise ratio and flow conditions and establish itself as a reliable uncertainty quantification algorithm for ensemble PIV.

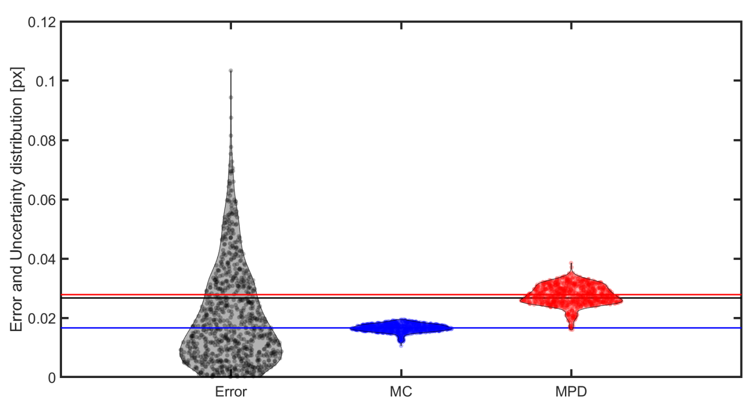

Figure 2: Error and Uncertainty distribution for the μPIV experiment.

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